

# RIEMANN

ExEuMetr: examples for →EuclM, Euclidean metric  [OK] gives example	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> <div>Ex: Euclidean Metric</div> <pre> Spherical: &lt; &lt; r θ ϕ &gt; Cylindrical: &lt; &lt; ρ ϕ z &gt; Circle S1: &lt; &lt; θ &gt; &lt; r &gt; Sphere S2 θ, ϕ: &lt; &lt; θ ϕ &gt; Sphere S2 r, ϕ: &lt; &lt; r ϕ &gt; Torus T2: &lt; &lt; θ ϕ &gt; &lt; r &gt; Sphere S3 r, θ, ϕ: &lt; &lt; r θ ϕ &gt; Sphere S3 x, θ, ϕ: &lt; &lt; x θ ϕ &gt; </pre> <p>CANCEL OK</p>	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> $\begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2(\theta) \end{bmatrix}$ <p>ExEuM +EuM ExGMe +GMet ExH+C H+Con</p>
→EuMetr: Euclidean metric of coordinates or manifold, ex: sph. coordinates (10s/0.5s) (HP50/Emulator on notebook) sphere S2 θ ϕ (4.5s/0.2)	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> $\begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2(\theta) \end{bmatrix}$ <p>ExEuM +EuM ExGMe +GMet ExH+C H+Con</p>	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> $\begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2(\theta) \end{bmatrix}$ <p>ExEuM +EuM ExGMe +GMet ExH+C H+Con</p>
→EuMetr: torus T2 (7.5s/0.2s)  sphere S3(14s/0.5s)	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> $\begin{bmatrix} (R+r\cos(\theta))^2 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2(\theta) \end{bmatrix}$ <p>ExEuM +EuM ExGMe +GMet ExH+C H+Con</p>	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> $\begin{bmatrix} R^2 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2(\theta) \end{bmatrix}$ <p>ExEuM +EuM ExGMe +GMet ExH+C H+Con</p>
ExGMetr: examples for →GMetr, general metric  spherical Minkowski (11s/0.5s)	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> <div>Ex: General Metric</div> <pre> Spherical Minkowski: &lt; Cylindrical Minkowski: &lt; Hyperbolic H2: &lt; &lt; r θ &gt; Hyperbolic H3: &lt; &lt; r θ &gt; Spherical de Sitter: &lt; Spher. Anti de Sitter: &lt; </pre> <p>CANCEL OK</p>	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r^2 \sin^2(\theta) \end{bmatrix}$ <p>ExEuM +EuM ExGMe +GMet ExH+C H+Con</p>
→GMetr: ex. hyperbolic H2 (5s/0.1s)  hyperbolic H3 (14s/0.6s)	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & L^2 + r^2 \end{bmatrix}$ <p>ExEuM +EuM ExGMe +GMet ExH+C H+Con</p>	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & L^2 + r^2 \end{bmatrix}$ <p>ExEuM +EuM ExGMe +GMet ExH+C H+Con</p>
→GMetr: spherical de Sitter (74s/3.5s)	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & L^2 + r^2 \sin^2(\theta) \end{bmatrix}$ <p>ExEuM +EuM ExGMe +GMet ExH+C H+Con</p>	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & L^2 + r^2 \sin^2(\theta) \end{bmatrix}$ <p>ExEuM +EuM ExGMe +GMet ExH+C H+Con</p>
ExMetric: examples coordinates, metric	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> <div>Ex: Metric +Conn, Geodes</div> <pre> Sphere S2 θ, ϕ: &lt; &lt; θ ϕ &gt; Sphere S2 r, ϕ: &lt; &lt; r ϕ &gt; Torus T2: &lt; &lt; θ ϕ &gt; &lt; r &gt; Hyperbolic H2 r, θ: &lt; Hyperbolic H2 x, θ: &lt; offdiagonal 2 dim: &lt; Sphere S3: &lt; &lt; r θ ϕ &gt; Hyperbolic H3: &lt; &lt; x θ ϕ &gt; </pre> <p>CANCEL OK</p>	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> <div>Ex: Metric +Conn, Geodes</div> <pre> Sphere S3: &lt; &lt; r θ ϕ &gt; Hyperbolic H3: &lt; &lt; x θ ϕ &gt; A, B(t, r): &lt; &lt; t r θ ϕ &gt; Spherical symmetry 4d Schwarzschild: &lt; &lt; t r θ ϕ &gt; Eddington-Finkelstein Kruskal-Szekeres: &lt; &lt; PRM: &lt; &lt; t r θ ϕ &gt; &lt; g &gt; </pre> <p>CANCEL OK</p>
M→Conn: metric to connection, (nonvanishing Christoffel symbols, note that $\Gamma^{\lambda\nu\mu} = \Gamma^{\lambda\mu\nu}$ is not shown and $\Gamma^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\nu\mu}$ ) sphere S2 r ϕ (22s/0.6s)	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> $\begin{bmatrix} 0 & 0 & 0 \\ 0 & R^2 - r^2 & 0 \\ 0 & 0 & r^2 \end{bmatrix}$ <p>ExEuM +EuM ExGMe +GMet ExH+C H+Con</p>	<pre> 0: 100: 200: 300: 400: 500: 600: 700: 800: 900: 1: </pre> $\begin{bmatrix} \Gamma^r_{rr} = \frac{r}{R^2 - r^2} \\ \Gamma^r_{\theta\theta} = -\frac{r(R^2 - r^2)}{R^2} \\ \Gamma^r_{\phi\phi} = -\frac{r}{R^2} \end{bmatrix}$ <p>ExEuM +EuM ExGMe +GMet ExH+C H+Con</p>

M→Conn: torus T2 (12s/0.6s)	4: 3: $g_{\mu\nu} = \begin{bmatrix} r^2 & 0 \\ 0 & (R+r\cos(\theta))^2 \end{bmatrix}$ $\begin{pmatrix} \theta \\ \rho \end{pmatrix}$ 1: $\begin{cases} \Gamma^{\theta\rho\rho} = \frac{(r\cos(\theta)+R)\sin(\theta)}{r} \\ \Gamma^{\theta\rho\rho} = \left[ -\frac{r\sin(\theta)}{r\cos(\theta)+R} \right] \end{cases}$ <div>ExEucl+EuclE3ExGHS+GHetEXH+C H+Con</div>	5: 4: 3: $g_{\mu\nu} = \begin{bmatrix} 1 & 0 \\ 0 & \sinh(\alpha)^2 \end{bmatrix}$ $\begin{pmatrix} x \\ \theta \end{pmatrix}$ 2: 1: $\begin{cases} \Gamma^{\theta\theta\theta} = (-\cosh(\alpha)\sinh(\alpha)) \\ \Gamma^{\theta\theta\theta} = \frac{\cosh(\alpha)}{\sinh(\alpha)} \end{cases}$ <div>ExEucl+EuclE3ExGHS+GHetEXH+C H+Con</div>
M→Conn: hyperbolic H2 (12s/0.4s)	2: 1: $g_{\mu\nu} = \begin{bmatrix} R^2 & 0 & 0 \\ R^2-r^2 & & \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\sin(\theta)^2 \end{bmatrix}$ <div>ExEucl+EuclE3ExGHS+GHetEXH+C H+Con</div>	1: $\begin{cases} \Gamma^{\theta\theta\theta} = \frac{1}{r} \\ \Gamma^{\theta\rho\rho} = (-\cos(\theta)\sin(\theta)) \\ \Gamma^{\theta\rho\rho} = \frac{1}{r} \\ \Gamma^{\theta\rho\rho} = \frac{\cos(\theta)}{\sin(\theta)} \end{cases}$ <div>ExEucl+EuclE3ExGHS+GHetEXH+C H+Con</div>
M→Conn: Schwarzschild (150s/3s)	3: 1: $g_{\mu\nu} = \begin{bmatrix} -\left(1-\frac{r_s}{r}\right) & 0 & 0 & 0 \\ 0 & \frac{1}{1-\frac{r_s}{r}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin(\theta) \end{bmatrix}$ <div>ExEucl+EuclE3ExGHS+GHetEXH+C H+Con</div>	1: $\begin{cases} \Gamma^{\theta\rho\rho} = (r_s-r)\sin(\theta)^2 \\ \Gamma^{\theta\rho\rho} = \frac{1}{r} \\ \Gamma^{\theta\rho\rho} = (-\cos(\theta)\sin(\theta)) \\ \Gamma^{\theta\rho\rho} = \frac{1}{r} \\ \Gamma^{\theta\rho\rho} = \frac{\cos(\theta)}{\sin(\theta)} \end{cases}$ <div>ExEucl+EuclE3ExGHS+GHetEXH+C H+Con</div>
M→Geodes: metric to geodesic ex: torus (23s/0.5s)	3: 2: 1: $g_{\mu\nu} = \begin{bmatrix} r^2 & 0 \\ 0 & (R+r\cos(\theta))^2 \end{bmatrix}$ $\begin{pmatrix} \theta \\ \rho \end{pmatrix}$ $\begin{cases} \theta'' + \rho'^2 \cos(\theta) \sin(\theta) + \frac{R\rho'^2}{r} \sin(\theta) \\ \rho'' - \frac{2\rho\rho'\theta'}{r\cos(\theta)+R} \sin(\theta) \end{cases}$ <div>H+Geo Geo+0 Geo+C ExC+R C+R+3 C+R+3</div>	4: 3: 2: 1: $g_{\mu\nu} = \begin{bmatrix} r^2 & 0 \\ 0 & r^2\sin(\theta)^2 \end{bmatrix}$ $\begin{pmatrix} \theta \\ \rho \end{pmatrix}$ $\begin{cases} \theta'' + \rho'^2 \cos(\theta) \sin(\theta) \\ \rho'' + \frac{2\rho\rho'\theta' \cos(\theta)}{\sin(\theta)} \end{cases}$ <div>H+Geo Geo+0 Geo+C ExC+R C+R+3 C+R+3</div>
Geo→DEQ: geodesic to differential equation sphere S2 (1s/0s)	4: 3: 2: 1: $\begin{cases} \theta'' + \rho'^2 \cos(\theta) \sin(\theta) \\ \rho'' + \frac{2\rho\rho'\theta' \cos(\theta)}{\sin(\theta)} \end{cases}$ $\begin{cases} d1d1\theta(s) - d1\rho(s)^2 \cos(\theta) \sin(\theta) \\ d1d1\rho(s) + \frac{2\rho d1\rho(s) d1\theta(s) \cos(\theta)}{\sin(\theta)} \end{cases}$ <div>H+Geo Geo+0 Geo+C ExC+R C+R+3 C+R+3</div>	2: 1: $\begin{cases} \theta'' + \rho'^2 \cos(\theta) \sin(\theta) + \frac{R\rho'^2}{r} \sin(\theta) \\ \rho'' - \frac{2\rho\rho'\theta'}{r\cos(\theta)+R} \sin(\theta) \end{cases}$ $\begin{cases} d1d1\theta(s) + d1\rho(s)^2 \cos(\theta) \sin(\theta) + \\ d1d1\rho(s) - \frac{2\rho d1\rho(s) d1\theta(s)}{r\cos(\theta)+R} \end{cases}$ <div>H+Geo Geo+0 Geo+C ExC+R C+R+3 C+R+3</div>
Geo→Conn: geodesic to connection ex: sphere S2 (2.3s/0s)	2: 1: $\begin{cases} \Gamma^{\theta\rho\rho} = \left( -\frac{r}{r^2-1} \right) \\ \Gamma^{\theta\rho\rho} = (r^2-r) \\ \Gamma^{\theta\rho\rho} = \frac{1}{r} \end{cases}$ <div>H+Geo Geo+0 Geo+C ExC+R C+R+3 C+R+3</div>	3: 2: 1: $\begin{cases} \Gamma^{\theta\rho\rho} = \frac{(r\cos(\theta)+R)\sin(\theta)}{r} \\ \Gamma^{\theta\rho\rho} = \left[ -\frac{r\sin(\theta)}{r\cos(\theta)+R} \right] \end{cases}$ <div>H+Geo Geo+0 Geo+C ExC+R C+R+3 C+R+3</div>
ExC→R: examples for connection to Riemann tensor	9: 8: 7: 6: 5: 4: 3: 2: 1: <div>Ex: Connection+Riemann Sphere S2 <math>\theta, \rho</math>: <math>\begin{cases} g_{\mu\nu} \end{cases}</math> Sphere S2 <math>r, \rho</math>: <math>\begin{cases} g_{\mu\nu} \end{cases}</math> Schwarzschild: <math>\begin{cases} g_{\mu\nu} \end{cases}</math> FRW: <math>\begin{cases} g_{\mu\nu} \end{cases}</math> [I '-1' 0 offdiagonal 2d: <math>\begin{cases} \begin{bmatrix} I &amp; I \end{bmatrix} \end{cases}</math> VAIDYA: <math>\begin{cases} \begin{bmatrix} I &amp; I \end{bmatrix} \end{cases}</math> -(1-2xH)</div> <div>CANCL OK</div>	6: 5: 4: 3: 2: 1: $\begin{cases} \Gamma^{\theta\rho\rho} = (-\cos(\theta)\sin(\theta)) \\ \Gamma^{\theta\rho\rho} = \frac{\cos(\theta)}{\sin(\theta)} \end{cases}$ <div>H+Geo Geo+0 Geo+C ExC+R C+R+3 C+R+3</div>
[OK] gives example		
C→Riemann: calculates components $R^\mu_{\nu\mu\nu}$ , ( $\mu \neq \nu$ ) of Riemann tensor for diagonal metric ex: S2 $\theta \phi$ (4s/0s) ex S2 $r \phi$ (4s/0s)	5: 4: 3: 2: 1: $\begin{cases} \Gamma^{\theta\rho\rho} = (-\cos(\theta)\sin(\theta)) \\ \Gamma^{\theta\rho\rho} = \frac{\cos(\theta)}{\sin(\theta)} \end{cases}$ $\begin{cases} R^{\theta\rho\rho\rho} = (\sin(\theta)^2) \\ R^{\theta\rho\rho\rho} = 1 \end{cases}$ <div>H+Geo Geo+0 Geo+C ExC+R C+R+3 C+R+3</div>	2: 1: $\begin{cases} \Gamma^{\theta\rho\rho} = \left( -\frac{1}{r^2-1} \right) \\ \Gamma^{\theta\rho\rho} = (r^2-r) \\ \Gamma^{\theta\rho\rho} = \frac{1}{r} \\ R^{\theta\rho\rho\rho} = (r^2) \\ R^{\theta\rho\rho\rho} = \frac{-1}{(r+1)(r-1)} \end{cases}$ <div>H+Geo Geo+0 Geo+C ExC+R C+R+3 C+R+3</div>



gRi→R: metric and Ricci tensor to Ricci scalar ex: sphere S2 (1s/0s)	5: 4: 3: $g_{\mu\nu} = \begin{bmatrix} r^2 & 0 \\ 0 & r^2 \sin^2(\theta) \end{bmatrix}$ 2: 1: $R_{\mu\nu} = \begin{bmatrix} 1 & 0 \\ 0 & \sin^2(\theta) \end{bmatrix}$ R: $\frac{2}{r^2}$ ExR→R R→Ric R→g <sub>uv</sub> gRi→R gRi→E TrHM	2: $\Gamma^{\alpha}_{\beta\gamma} = \begin{bmatrix} \Gamma^r_{rr} = \frac{1}{r} \\ \Gamma^r_{\theta\theta} = (r^2 - r) \\ \Gamma^{\theta}_{r\theta} = \frac{1}{r} \\ \Gamma^{\theta}_{\theta r} = (r^2) \\ \Gamma^{\theta}_{\theta\theta} = \frac{-1}{(r+1)(r-1)} \end{bmatrix}$ 1: H→Geo Geo→D Geo→C ExC→R C→Ric C→g <sub>uv</sub>
M→Conn: offdiagonal Minkowski (120s/2.6s)	4: 3: 2: 1: $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1+u^2 & -u+2uv \\ 0 & 0 & -u+2uv & 1+u^2 \end{bmatrix}$ ExEuc→EucU ExHM→HMNH ExH3→H3Con	4: 3: $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1+u^2 & -u+2uv \\ 0 & 0 & -u+2uv & 1+u^2 \end{bmatrix}$ 2: 1: $\Gamma^{\alpha}_{\beta\gamma} = \begin{bmatrix} \Gamma^u_{uu} = \frac{u^2 u_{,u} + 2u}{(u^2 u^2 + u^2)_{,u} + 4u_{,u} - (4u^2 - 1)} \end{bmatrix}$ ExEuc→EucU ExHM→HMNH ExH3→H3Con
C→Riemann: offdiagonal Minkowski (70s/0.5s)  R→Ricci: (29s/0.6s)	1: $\begin{bmatrix} R_{uuuu} = \frac{(1-4u^2+4u_{,u}+(u^2+u^2 u^2))}{(1+u^2)(4-u)u} \\ R_{uuuu} = \frac{(1-4u^2+4u_{,u}+(u^2+u^2 u^2))}{(2(u-u)(4-u)u} \\ R_{uuuu} = \frac{(1-4u^2+4u_{,u}+(u^2+u^2 u^2))}{(1-4u^2+4u_{,u}+(u^2+u^2 u^2))} \end{bmatrix}$ H→Geo Geo→D Geo→C ExC→R C→Ric C→g <sub>uv</sub>	2: 1: $\begin{bmatrix} R_{\mu\nu} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$ ExR→R R→Ric R→g <sub>uv</sub> gRi→R gRi→E TrHM
MRi→R: metric and Ricci tensor to Ricci scalar (32s/0.6s). For a=4 R=0.  FLRW metric	2: 1: $R = \frac{(a-4)u.2}{((u^2 u^2 + u^2)_{,u} + 4u_{,u} - (4u^2 - 1))^2}$ ExR→R R→Ric R→g <sub>uv</sub> gRi→R gRi→E TrHM	2: 1: $g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \frac{a(t)^2}{1-kr^2} & 0 \\ 0 & 0 & a(t)^2 r^2 \end{bmatrix}$ ExR→R R→Ric R→g <sub>uv</sub> gRi→R gRi→E TrHM
FLRW Ricci tensor  gRi→E: metric and Ricci tensor to Einstein tensor Ex: FRW (43s/1s)	2: 1: $R_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{a(t) \cdot d \cdot d \cdot a(t)}{r^2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$ ExR→R R→Ric R→g <sub>uv</sub> gRi→R gRi→E TrHM	2: 1: $G_{\mu\nu} = \begin{bmatrix} \frac{3 \cdot d \cdot a(t)^2 + 2 \cdot k}{a(t)^2} \\ 0 & \frac{2 \cdot a(t) \cdot d \cdot d \cdot a(t)}{r^2} \\ 0 & 0 \end{bmatrix}$ ExR→R R→Ric R→g <sub>uv</sub> gRi→R gRi→E TrHM
ExGT→Eeq: example Einstein G and energy mom. tensor T to Einstein equations ex: FRW  GT→Eeq (6s/0s)	2: 1: $T_{\mu\nu} = \begin{bmatrix} \rho(t) & 0 & 0 \\ 0 & \frac{p \cdot a(t)^2}{1-kr^2} & 0 \\ 0 & 0 & p \cdot a(t)^2 r^2 \end{bmatrix}$ ExGT→GT→Eeq HelpR	1: $\begin{bmatrix} \frac{d \cdot a(t)^2 + k}{a(t)^2} \\ \frac{2 \cdot a(t) \cdot d \cdot d \cdot a(t) + d \cdot a(t)}{r^2 \cdot k - 1} \\ -((2 \cdot a(t) \cdot d \cdot d \cdot a(t) + d \cdot a(t)^2 + k) \cdot r^2) \end{bmatrix}$ ExGT→GT→Eeq HelpR PPAR
1. equation (before and after simplifying)  2. equation (3. and 4. eq are identical)	4: 3: 2: 1: $\frac{(d \cdot a(t)^2 + k) \cdot 2}{a(t)^2} = k \cdot \rho(t)$ List ExEuc→EucU ExHM→HMNH ExH3→H3Con	4: 3: 2: 1: $\frac{2 \cdot a(t) \cdot d \cdot d \cdot a(t) + d \cdot a(t)^2 + k}{r^2 \cdot k - 1} = k \cdot \frac{-p}{r^2}$ List ExEuc→EucU ExHM→HMNH ExH3→H3Con
M→Ricci: diagonal metric to Ricci tensor ex: sphere S2 (26s/0.6s)  ex: torus T2 (40s/1s)	3: 2: 1: $g_{\mu\nu} = \begin{bmatrix} \frac{1}{1-r^2} & 0 \\ 0 & r^2 \end{bmatrix}$ $R_{\mu\nu} = \begin{bmatrix} -1 & 0 \\ 0 & r^2 \end{bmatrix}$ ExGT→GT→Eeq H→R R→Ric gRi→R gRi→E TrHM	4: 3: 2: 1: $g_{\mu\nu} = \begin{bmatrix} r^2 & 0 \\ 0 & (R+r \cos(\theta))^2 \end{bmatrix}$ $R_{\mu\nu} = \begin{bmatrix} \frac{r \cos(\theta)}{r \cos(\theta) + R} & 0 \\ 0 & \frac{(r \cos(\theta) + R) \cdot C}{r} \end{bmatrix}$ ExGT→GT→Eeq H→R R→Ric gRi→R gRi→E TrHM

[illegible]

HelpRIEMANN: help to programs	M+Ricci Metric + Riemann tensor {x^2} [[g <sub>μν</sub> ]] + [[g <sub>μν</sub> ]] {x^2} [[R <sub>μν</sub> ]] M+R Metric + Ricci tensor {x^2} [[g <sub>μν</sub> ]] + R dM+Kret Metric + Ricci scalar {x^2} [[g <sub>μν</sub> ]] + R diagonal Metric to R = R <sub>μννμ</sub> R <sup>μνμν</sup> Kretschmann scalar displays singularities dM+T <sub>μν</sub> [[g <sub>μν</sub> ]] + [[g <sub>μν</sub> ]] [[T <sub>μν</sub> ]] +SKIP SKIP+ +DEL DEL+ DEL L INS = Collect [[t3] + [[t3]	dM+T <sub>μν</sub> [[g <sub>μν</sub> ]] + [[g <sub>μν</sub> ]] [[T <sub>μν</sub> ]] diagonal Metric in 4 dim to energy momentum tensor of fluid in comoving coordinates (u4)A=-3a0 u1=0, ex:FLRW 'd2d1g <sub>μν</sub> (r,θ)' + '30(3r(g <sub>μν</sub> (r,θ)))' works for d1..d3 TriHyp [[t3] + [[t3] simplify terms with SIN(θ),COS(θ) Collect [[t3] + [[t3] +SKIP SKIP+ +DEL DEL+ DEL L INS =
HelpRIEMANN: help to programs	Collect [[t3] + [[t3] Collect FOR MATRIX,LIST Fdistrib [[t3] + [[t3] FDISTRIB FOR MATRIX,LIST Eval [[t3] + [[t3] EVAL FOR MATRIX,LIST Subst [[t3] <'v1=v1'..3 SUBSTITUTE VARS PERFORMS d+der Match [[t3] <t1 t23 + [[t3] <t3' Ex: RIEMANN TENSOR OF Eddington Finkelstein METRIC OBTAINED WITH M+Riemann, SUBSTITUTE <'σ^2' 12 d2d1+d1d2 [[t3] + [[t3] replace d2d1g(t,r) + d1d2g(t,r) and COLLECT 4 +SKIP SKIP+ +DEL DEL+ DEL L INS =	Substitute VARS PERFORMS d+der Match [[t3] <t1 t23 + [[t3] <t3' Ex: RIEMANN TENSOR OF Eddington Finkelstein METRIC OBTAINED WITH M+Riemann, SUBSTITUTE <'σ^2' 12 d2d1+d1d2 [[t3] + [[t3] replace d2d1g(t,r) + d1d2g(t,r) and COLLECT 4 +SKIP SKIP+ +DEL DEL+ DEL L INS =

The software RIEMANN provides tools for calculating metric, connection, geodesic equations, Riemann and Ricci tensor and Einstein equations step by step. The programs run rather fast on the HP50 emulator on a computer or tablet. The software requires the MPC font for correct display of tensor indices.

Some important books on the subject are:

- A. Zee, Einstein Gravity in a Nutshell
- L. Ryder, Introduction to General Relativity
- M. Nakahara, Geometry, Topology and Physics