

# PADE

<p>Pade: Pade approximant of SIN(X) (4s emulator)</p> <p>Graph of P10,10 and sin(x)</p>	<pre> 7: "Pade" 6: SIN(X) 5: X0:0 4: n:10 3: n:10 2: 35143 1: 139754432 P10,10: 25802136829 153729882268441839360 ExPad Pad PadSq PadSF GrFX FX0 </pre>	
<p>Pade: numeric Pade approximant of COS(X) (18s)</p> <p>Graph of P25,25 and COS(X)</p>	<pre> 7: "Pade NUM." 6: COS(X) 5: X0:0 4: n:25 3: n:25 2: 25 1: 2.61256812425E-43 P25,25: 2.51646323678E-51 ExPad Pad PadSq PadSF GrFX FX0 </pre>	
<p>Pade: Pade approximant of COS(X) around <math>\pi/4</math> (8s)</p> <p>Graph of P5,8 and COS(X)</p>	<pre> 6: COS(X) 5: X0:pi/4 4: n:5 3: n:8 2: 8 1: 51575502798257 269361021172953790080 ExPad Pad PadSq PadSF GrFX FX0 </pre>	
<p>PadSer: Pade approximant of LN(1+X)/X around <math>\pi/4</math> with SERIES command (7s)</p> <p>Graph of P8,8 and LN(1+X)/X</p>	<pre> 6: LN(1+X) 5: X 4: X0:0 3: n:8 2: n:8 1: 218790 P8,8: 218790 24310 ExPad Pad PadSq PadSF GrFX FX0 </pre>	
<p>PadSF: numeric Pade approximant of LambW(X) around 1 (26s)</p> <p>Graph of P4,8 and LambW(X)</p>	<pre> 7: "PadSF" 6: LambW(X) 5: 1 4: 4 3: 4 2: 4 1: 9.86403679879E-3 P4,4: 9.86403679879E-3 2.95930895449E-3 ExPad Pad PadSq PadSF GrFX FX0 </pre>	
<p>anPade: Pade approximant of divergent sequence (2s)</p> <p>IPade: Pade approximant of list (2s)</p>	<pre> 7: an: (-1)^n * n! 6: n:5 5: n:6 4: 1764 P5,6: 1764 720 GrRes ExanP anPad ExIPa IPade an+L </pre>	<pre> 4: -6227020800 87178291200 -1307674368000 20922789888000 3: 8 2: 8 1: 40320 P8,8: 40320 362880 GrRes ExanP anPad ExIPa IPade an+L </pre>
<p>TAYLN: Taylor series of COS(X) (1s)</p> <p>TAYSer: Taylor series of LN(1+X)/X (1s)</p>	<pre> 7: "TAYLN" 6: COS(X) 5: pi/3 4: 3 3: 13 2: 13 1: 1/2 * sqrt(3) * (x - pi/3) + 1/4 * (x - pi/3)^2 + sqrt(3)/12 * (x - pi/3)^3 ExTAY TAYLN TAYSe TAY+SF+SEQ -dum </pre>	<pre> 7: "TAYSer" 6: LN(1+X) 5: X 4: 0 3: 0 2: 20 1: -1/20 * x^19 + 1/19 * x^18 - 1/18 * x^17 + 1/17 * x^16 + ... ExPad Pad PadSq PadSF GrFX FX0 </pre>

<p>TAYLN: Graph of Taylor series of Si(X) and function</p> <p>f-&gt;SEQ: Taylor sequene of Si(X) up to 20 (3s)</p> <p>-&gt;Num: numeric values (0.1s)</p>		<pre> 8: 7: 6: 5: 4: 3: 2: 1: {Si(1) SIN(1) - SIN(1) - COS(1) ST 2 1: 2.946083070367 .841470984808 ExTAY TAYLN TAYSe TAY+S F+SEQ +NUM </pre>
<p>f-&gt;SEQ: Taylor sequene of Erf(X) to 20 (3s). (Only when derivatives don't blow up!)</p> <p>After Erf(0)=0 with lPade: exact Pade 10 10 (9s)</p>		<pre> 8: 7: 6: 5: 4: 3: 2: 1: {0 2*sqrt(n) 0 -2*sqrt(n) 0 sqrt(n) 0 -sqrt(n) n n 3*n 5*n 7*n 1: 2642839043372350 7729742959889212 P10,10: 458206790715157 22943840214274327920 GrRes ExanP anPad ExLPa LPade an+1 </pre>
<p>GrfX: graph of P10,10</p> <p>GrfX: graph of Erf(X)</p>		
<p>HelpPade: Help to programs</p>	<p>PADE APPROXIMATION OF F(X)=Ean*X^n, an, {a0..an} BY RATIONAL FUNCTIONS PNM(X) PNM: (A0*X^n+...+A0)/(B0*X^n+...+B0)</p> <p>ExPade _ + F(X) NO n M EXAMPLES FOR Pade, PadSer, PadSF</p> <p>Pade F(X) NO n M + F(X) NO PNM(X) PADE APPROXIMANT</p> <p>PadSer F(X) NO n M + F(X) NO PNM(X) PADE APPROXIMANT WITH SERIES COMMAND</p> <p>+SHIP SHIP+ +DEL DEL+ DEL L INS=</p>	<p>PadSF F(X) NO n M + F(X) NO PNM(X) NUMERIC PADE FOR SPECIAL FUNCTIONS</p> <p>TAY FIRST WITH LOW n M</p> <p>F(X) NO, CH1..3 + F(X) P(X) NO, CP(X)1..3</p> <p>GrFX F(X) + F(X) GRAPH</p> <p>GrReset _ + _ RESET FCT PLOT</p> <p>4 To (-10,-6) (10,6)</p> <p>ExanPade _ + an n M EXAMPLES</p> <p>+SHIP SHIP+ +DEL DEL+ DEL L INS=</p>
<p>HelpPade: Help to programs</p>	<p>ExanPade _ + an n M EXAMPLES FOR anPade</p> <p>anPade an n M + an PNM</p> <p>ExLPade _ + {a1..an} EXAMPLES FOR LPade</p> <p>LPade {a1..an} n M + {3} PNM</p> <p>an+1 _ + a(n) n1 n2 + a(n) {a(n1)..a(n2)} SEQUENCE TO LIST</p> <p>4 EXTAYL _ + F(X) NO n</p> <p>+SHIP SHIP+ +DEL DEL+ DEL L INS=</p>	<p>EXTAYL _ + F(X) NO n</p> <p>TAYLN F(X) NO n + F(X) NO E(j)=0,n,a(j)*X^j TAYLOR SERIES UP TO n</p> <p>TAYser F(X) NO + F(X) NO E(j)=0,n,an*X^n TAYLOR SERIES UP TO 21 WITH SERIES COMMAND</p> <p>TAY+SEQ E(j)=0,n,an*X^n NO + NO {a0..an} SERIES TO SEQUENCE</p> <p>+SHIP SHIP+ +DEL DEL+ DEL L INS=</p>
<p>HelpPade: Help to programs</p>	<p>f+SEQ F(X) NO n + F(X) NO {a0..an} FUNCTION TO SEQUENCE</p> <p>+NUM 0 + 0' NUMERICAL APPROX. OF TERM, LIST</p> <p>SDerVX F(X) + 3F(X) DERIVATIVE OF SPECIAL FUNCTIONS</p> <p>GrpFX F(X) + F(X) GRAPH WITH POINTS</p> <p>4 PROGRAMS USE DIR AND LIB SFUNC</p> <p>+SHIP SHIP+ +DEL DEL+ DEL L INS=</p>	<p>GrpFX F(X) + F(X) GRAPH WITH POINTS</p> <p>PROGRAMS USE DIR AND LIB SFUNC</p> <p>HINT: IN APPROXIMATE MODE WITH ELS)8(CENTER) YOU GET NUMERIC COEFFICIENTS (FASTER).</p> <p>+SHIP SHIP+ +DEL DEL+ DEL L INS=</p>
<p>InfoPade: short Info to Pade</p>	<p>SHORT INFO ON PADE APPROXIMATION</p> <p>4 (DIVERGENT) POWER SERIES CAN OFTEN BE APPROXIMATED WITH PADE ALSO IF ONLY A FEW TERMS ARE KNOWN. ONE DETERMINES Ean*X^n (n=0..N) BY A SEQUENCE OF RATIONAL FUNCTIONS PNM(X)=E(n=0,N,An*X^n) / E(n=0,N,Bn*X^n) WITH THE EQUATIONS</p> <p>+SHIP SHIP+ +DEL DEL+ DEL L INS=</p>	<p>[E1..BM]=-[a0+1..an+M]/[1ai,j] [1ai,j]=[a0+i-j] (M*N MATRIX)</p> <p>[An]=E(j=0,n,an-j*Bj) 0≤n≤N, Bj=0 FOR j&gt;N.</p> <p>THIS IS OBTAINED FROM Eaj*X^j=P(X)/Q(X) OR E(j=0,N,M,a(j)*X^j)*E(k=0,M,Bk*X^k)=E(n=0,N,An*X^n).</p> <p>+SHIP SHIP+ +DEL DEL+ DEL L INS=</p>
<p>InfoPade: short Info to Pade</p>	<p>OFTEN ONE USES PNM, PNM+1 AS APPROXIMANTS. THE PROGRAMS CALCULATE THEM AND YOU CAN INSERT X VALUES. FOR STIELTJES FUNCTIONS OF THE FORM Ean*(-z)^n (WITH SOME EXTRA PROPERTIES), PNM DECREASE AND PNM+1 INCREASE MONOTONICALLY WITH n, GIVING AN ERROR OF THE APPROXIMATION. IN THIS CASE PNM, PNM+1 CONVERGE IN THE COMPLEX CUT PLANE, BRANCH CUT IS THE NEGATIVE AXIS.</p> <p>+SHIP SHIP+ +DEL DEL+ DEL L INS=</p>	<p>MONOTONICALLY WITH n, GIVING AN ERROR OF THE APPROXIMATION. IN THIS CASE PNM, PNM+1 CONVERGE IN THE COMPLEX CUT PLANE, BRANCH CUT IS THE NEGATIVE AXIS.</p> <p>LITERATURE: C.M.BENDER, S.A.ORSZAG ADVANCED MATHEMATICAL METHODS FOR SCIENTISTS AND ENGINEERS SPRINGER 1999</p> <p>+SHIP SHIP+ +DEL DEL+ DEL L INS=</p>